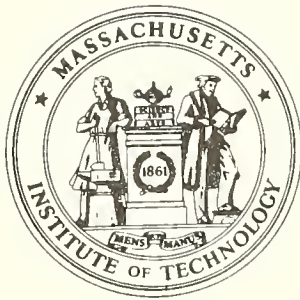


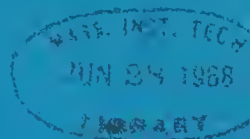


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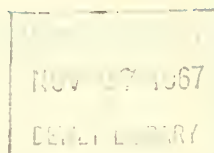
COMMENT

by

Peter A. Diamond

Number 4 -- October 1967

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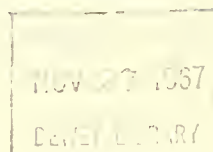
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Introduction

In considering rules for public investment to maximize a social welfare function, Stephen A. Marglin has argued^{1/} for calculations based on a discount rate reflecting the social rate of discount (presumably below the market rate of return) combined with a shadow price of capital in excess of one. The form of the argument was to construct a simple model presenting the basic justification for this result and then to introduce various complications which did not interfere with its validity. The approach here will be the reverse, to start with a very general model and introduce assumptions which lead to the same conclusion. This approach will place in perspective the assumptions leading to the conclusion, for these assumptions appear to the author to be too restrictive to be employed as a basis for policy.

* The author is indebted to Stephen Marglin for helpful discussion on this topic. During part of the work on this comment he was an Overseas Fellow at Churchill College, Cambridge. Financial support was provided in part by the National Science Foundation under grant GS 1585. The author alone is responsible for errors and opinions.

1/ "The Opportunity Costs of Public Investment," Quarterly Journal of Economics, LXXVII, No. 2 (May, 1963). Some of these comments are also relevant for M. S. Feldstein, "Net Social Benefit Calculation and the Public Investment Decision," Oxford Economic Papers, 16 (March, 1964).

General Formulation

We are considering a government making a once-for-all production decision which will maximize a social utility function, u , which is a function of an (infinite-dimensional) vector of consumption, x , where the individual arguments reflect consumption of each good in each period by each consumer. Let z be the vector of net government production (outputs are measured positively and inputs negatively). We shall write the government production constraint as

$$1. \quad g(z) = 0.$$

In addition to public production, there is private production, which we denote by the vector y . The equilibrium quantities of private production depend on the vector of public production quantities,

$$2. \quad y = y(z).$$

(Thus each y_i depends on all z_i .) Presumably the form of the vector function y depends in part on other government decisions, e.g., taxation, which are made in response to the production decision.

Since consumption equals the sum of public and private production,

$$3. \quad x = y + z,$$

we can state the government maximization problem as

$$4. \quad \begin{array}{l} \text{Maximize } u(z + y(z)) \\ \text{subject to } g(z) = 0. \end{array}$$

The first order conditions for this maximization are:

$$5. \quad u_j + \sum_i u_i \frac{\partial y_i}{\partial z_j} + \lambda g_j = 0 \quad j = 1, 2, \dots$$

where λ is a Lagrange multiplier and u_j the derivative of u with respect to its j^{th} argument. Taking two first order conditions, we obtain the marginal rate

of transformation for optimal government production:

$$6. \quad \frac{g_j}{g_k} = \frac{u_j + \sum_i u_i \frac{\partial y_i}{\partial z_j}}{u_k + \sum_i u_i \frac{\partial y_i}{\partial z_k}}.$$

Without simplifying assumptions, this general formulation is far too complicated to be employed as an investment rule. However there are several elements of the formulation which seem worth noting. As has been argued by Marglin and others, benefits should be discounted at a social marginal rate of substitution and the benefits which are to be discounted are not just government produced benefits but also the change in privately produced benefits arising from government production. The summation of terms reflecting the change in private production includes all production variables. While some of these derivatives may be zero, we would expect many of them not to be. In particular we can expect government production to affect private production which occurs even before the public benefits, a case unfortunately excluded when savings depend on current income independent of expectations about the future.

Two Period Case

The simplest case which naturally arises in a search for special cases is that of a two period model. This case furthermore gives an intuitively appealing result, but one which, I feel, is extremely misleading in the more general case. Let us assume that there are only two goods, consumption today and tomorrow, and that the social utility function is a function of aggregate consumption in each period. We can restate the maximization as

$$7. \quad \begin{aligned} &\text{Maximize } u(z_1 + y_1(z_1, z_2), z_2 + y_2(z_1, z_2)) \\ &\quad z_1, z_2 \\ &\text{subject to } g(z_1, z_2) = 0. \end{aligned}$$

Let us denote private production possibilities as

$$8. \quad f(y_1, y_2) = 0.$$

Then the derivatives of private with respect to public production are constrained to satisfy

$$9. \quad f_1 \frac{\partial y_1}{\partial z_i} + f_2 \frac{\partial y_2}{\partial z_i} = 0 \quad i = 1, 2,$$

where $f_i = \frac{\partial f}{\partial y_i}$.

Considering z_2 as a function of z_1 (the public production constraint), there is one decision variable, the level of public investment in the first period, which equals $-z_1$. We can then consider the total general equilibrium response of present privately produced consumption to public investment.

$$10. \quad \frac{dy_1}{dz_1} = \frac{\partial y_1}{\partial z_1} - \frac{g_1}{g_2} \frac{\partial y_1}{\partial z_2}.$$

The first order condition for public investment is (equation (6) reduced to the two period case):

$$11. \quad \frac{g_1}{g_2} = \frac{u_1 + u_1 \frac{\partial y_1}{\partial z_1} + u_2 \frac{\partial y_2}{\partial z_1}}{u_2 + u_1 \frac{\partial y_1}{\partial z_2} + u_2 \frac{\partial y_2}{\partial z_2}}.$$

With appropriate manipulation, this can be shown to be equivalent to

$$12. \quad \frac{g_1}{g_2} = (1 + \frac{dy_1}{dz_1}) \frac{u_1}{u_2} - \frac{dy_1}{dz_1} \frac{f_1}{f_2}.$$

Noting that the response of private investment to current government benefits is $-\frac{dy_1}{dz_1}$ while the response of consumption is $1 + \frac{dy_1}{dz_1}$, equation (12) states that the margin for optimal public investment is a weighted average of private marginal rates of substitution and transformation, the weights being the equilibrium responses of private consumption and investment. Let us note that these latter derivatives reflect not just the impact of public investment on

present consumption (for example by taxes used for finance) but also the impact of future benefits on present consumption (for example by the realization by consumers that part of their future is provided for).

Although equation (12) is intuitively very appealing, it must be reiterated that the equation is misleading. In the two period formulation, all public investments and private investments have the same time path of future benefits. In the general case, not only don't all investments have the same benefit stream, but the margins of investment are needed to choose the shape of the benefit stream coming from a particular investment. The next section offers an example which is counter to a naive extension of the two period case, where present public investment falls on consumption and private investment alike but the optimal margin is the marginal rate of substitution.

Stationarity

Let us continue to assume that there is but one consumption good in each period and that social welfare is a function of aggregate consumption levels. However we return to the general case of an unbounded time horizon.

The term stationarity is used to refer to comparisons made of an economy at two different dates which depend solely on the lapsed time between the dates, not on calendar time. We shall make two specific assumptions of this nature, one on the derivatives of the social utility function; the other on the interactions of public and private investment.

We assume that social marginal utilities decrease over time at a geometric rate,

$$13. \quad u_i = a(1 + r)^{-i}$$

where a is a constant; and r the (constant) interest rate implicit in the social marginal rate of substitution.

The second assumption is that the response of privately produced consumption to publicly produced consumption depends solely on the time lapse (perhaps negative) between the two dates

$$14. \quad \frac{\partial y_i}{\partial z_j} = \phi(i - j)$$

where ϕ is a function defined over the integers. To reach our conclusion it is only necessary for (13) and (14) to hold at the optimal configuration of the economy not necessarily for any position of the economy.

Let us note that Marglin assumed (13) directly. The assumptions of a fixed savings fraction and fixed returns (constant over time) to both public and private investment, imply (14). These assumptions were made by Marglin for all periods but the present, so (14) was implied for all i and for $j = 1, 2, \dots$ (noting the present by the subscript 0).

To begin, let us assume that (14) holds for all i and for j equal to two values, s and t , $s < t$. Since the present cannot affect the past, $\frac{\partial y_i}{\partial z_s}$ must be zero for i negative, or, with (14),

$$15. \quad \phi(h) = 0 \quad \text{for } h < -s.$$

This, in turn puts a restriction on the effects z_t can have on the years from the present until $t - s$.

$$16. \quad \frac{\partial y_i}{\partial z_t} = 0 \quad \text{for } i < t - s.$$

Thus there is a restriction on the effects of public action on private action taken in anticipation of public action. If, as Marglin does, one assumes that (14) holds for all future public investment ($j > 0$), no anticipatory action can be taken more than one period in advance. This seems to be a serious implication, ruling out, for example, life-cycle savings, where future social security benefits, for example, affect present consumption.

With these two assumptions we can now show that the margin for optimal investment is the social marginal rate of substitution.

$$\begin{aligned}
 17. \quad \frac{g_s}{g_t} &= \frac{a(1+r)^{-s} + a \sum_{i=0}^{\infty} (1+r)^{-i} \phi(i-s)}{a(1+r)^{-t} + a \sum_{i=0}^{\infty} (1+r)^{-i} \phi(i-t)} \\
 &= \frac{(1+r)^{-s} (1 + \sum_{i=0}^{\infty} (1+r)^{s-i} \phi(i-s))}{(1+r)^{-t} (1 + \sum_{i=0}^{\infty} (1+r)^{t-i} \phi(i-t))}.
 \end{aligned}$$

Define two new indices, $j = i-s$ and $k = i-t$. Then, we can write

$$18. \quad \frac{g_s}{g_t} = (1+r)^{t-s} \frac{1 + \sum_{j=-s}^{\infty} (1+r)^{-j} \phi(j)}{1 + \sum_{k=-s}^{\infty} (1+r)^{-k} \phi(k) + \sum_{k=-t}^{-s-1} (1+r)^{-k} \phi(k)}.$$

But by (15), the second sum in the denominator is zero and we have

$$19. \quad \frac{g_s}{g_t} = (1+r)^{t-s}.$$

Thus, assuming that (14) holds for the entire future, the optimal margins for public investment are found by using a discount rate implicit in the social rate of time preference, and a shadow price of present investment different from one (since (19) does not hold for $t = 0$).

Assumptions (13) and (14) were both needed to construct this proof, although alternate pairs of assumptions will presumably lead to the same conclusion. It is perhaps useful to reiterate the importance of anticipations in the economy (and so, the severity of these assumptions). With consumers maximizing lifetime utility functions, a dollar of taxes today which leads to no future benefits falls on consumption and saving in a way reflected by the marginal propensity to save. However, a dollar of taxes today which gives rise to (fully expected) future benefits which reflect the private rate of

return falls fully on present savings. Only insofar as the rate of return, implied by the benefits arising from the taxes, differs from the private rate of return do taxes fall on present consumption. Of course, this picture is complicated by uncertainty, differences between beneficiaries and tax payers, and the non-marketability of many government benefits. Thus, using a model where the time path of future benefits has no effect on current savings seems ill-suited for examining the discount rate for public investment.

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